



**Fig. S2 Full model comparison.** (A) HIV/HPV co-infection model comparison (a) HPV  $W$  and (b) CTL  $E$  as given by model (5), solid blue lines and model (26), dashed red lines, for  $\varepsilon = 0.5$  per day, parameters are listed in Table 1 for different  $\bar{T}$  levels-  $\bar{T} = 10^6$  cells per ml (first row);  $\bar{T} = 5 \times 10^5$  cells per ml (second row);  $\bar{T} = 3.3 \times 10^5$  cells per ml (third row); and  $\bar{T} = 2 \times 10^5$  cells per ml (fourth row). (B) HIV/HPV dynamics when cART and HPV infection coincide (a) HPV  $W$ ; (b) CD4+ T cells ( $T$ ) as given by model (5) solid blue lines and model (26) dashed red lines under cART. Here,  $\varepsilon = 0.5$ ,  $\varepsilon_{RT} = 0.95$ ,  $\varepsilon_{PI} = 0.5$ , and all other parameters are listed in Table 1 and  $t = 0$  is the start of cART. Over the first 24 weeks HPV persists (panel a), and in the long term HPV is cleared (zoomed out panel a).